Loopy Lévy flights enhance tracer diffusion in active suspensions

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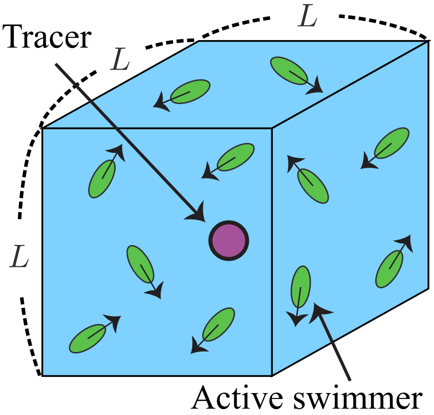
Passive tracer diffusing in an active suspension

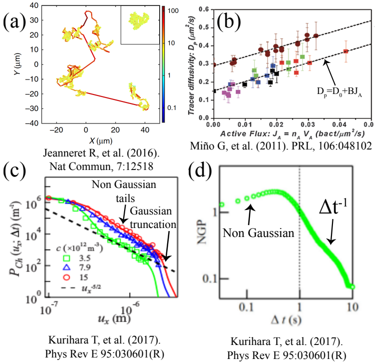


- Low Reynolds number swimming
  - *Volvox*
  - *Chlamydomonas*
  - *E. coli*

- Low density

Tracer diffusion in active suspensions: Experimental results

Characteristic features	
Loopy trajectories	(a)
Enhanced diffusion	(b)
Non-Gaussian tails of the displacement statistics	(c)
Reversion to Gaussian	(c)
NGP $\simeq \Delta t^{-1}$	(d)



Jeanneret R, et al. (2016). Nat Commun, 7:12518

Miño G, et al. (2011). PRL, 106:048102

Kurihara T, et al. (2017). Phys Rev E 95:030601(R)

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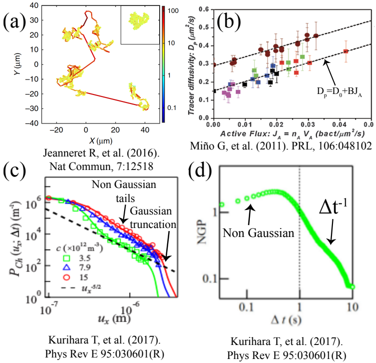
Tracer diffusion in active suspensions: Experimental results

Characteristic features		T1	T2
Loopy trajectories	(a)	✓	✗
Enhanced diffusion	(b)	✓	✗
Non-Gaussian tails of the displacement statistics	(c)	✗	✓
Reversion to Gaussian	(c)	✗	✓
NGP $\simeq \Delta t^{-1}$	(d)	✗	✗



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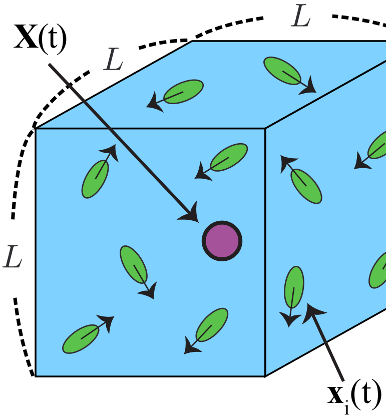
T1: Phenomenological "active flux": Jepson A, et al. (2013). Phys Rev E, 88:041002(R)

T2: Holtsmark-type theory: Zaid & Mizuno (2016). Phys Rev Lett 117:030602

Microscopic model for swimmer–tracer system



- Equations of motion:

$$\frac{d\boldsymbol{x}_i}{dt} = v_{\mathrm{A}}\boldsymbol{n}_i$$

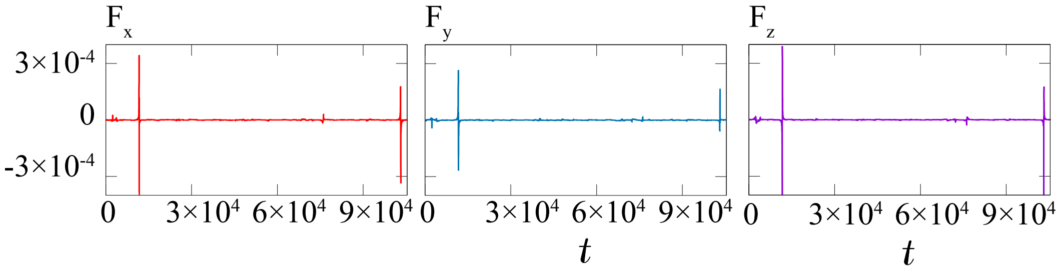
$$\Gamma\frac{d\boldsymbol{X}}{dt} = \sum_{i=1}^{N}\boldsymbol{F}(\boldsymbol{r}_i,\boldsymbol{n}_i) \qquad \boldsymbol{r}_i \equiv \boldsymbol{x}_i - \boldsymbol{X}$$

- Truncated hydrodynamic force (dipole):

$$\boldsymbol{F} \equiv \begin{cases} \frac{p}{r_i^2}\left[3\frac{(\boldsymbol{n}_i\cdot\boldsymbol{r}_i)^2}{r_i^2} - 1\right]\frac{\boldsymbol{r}_i}{r_i} & r_i > d \\ 0 & r_i \leq d \end{cases}$$

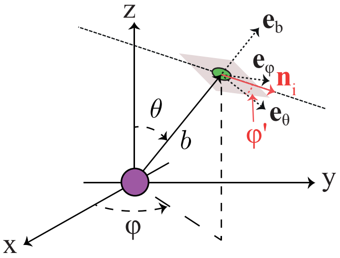
- In dilute conditions ($\rho \ll 1/d^3$) only dipolar far-flow field relevant

Force dynamics: independent kicks



- In dilute conditions tracer dynamics dominated by two-body scattering events
- $\boldsymbol{F}$=sum of random scattering events
- Map multi-particle dynamics onto simpler stochastic process

Langevin dynamics: coloured Poisson noise



- Exact equations of motion:

$$\frac{d\boldsymbol{x}_i}{dt} = v_{\rm A}\boldsymbol{n}_i, \qquad \Gamma\frac{d\boldsymbol{X}}{dt} = \sum_{i=1}^{N}\boldsymbol{F}(\boldsymbol{r}_i, \boldsymbol{n}_i)$$

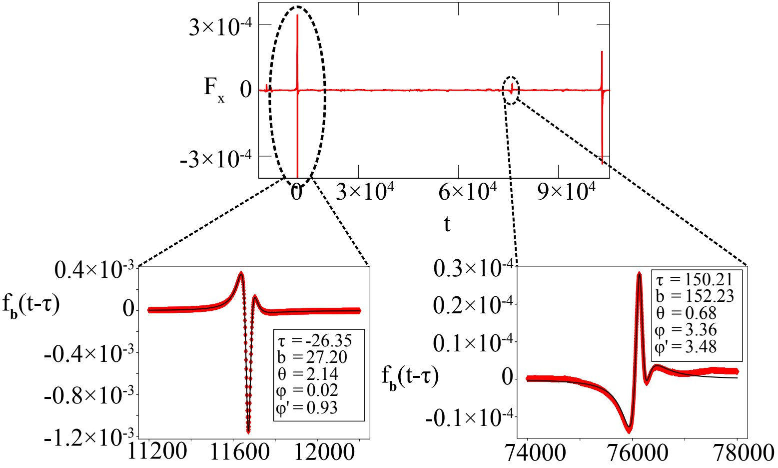
Map exact dynamics on coloured Poisson process

- Langevin dynamics:

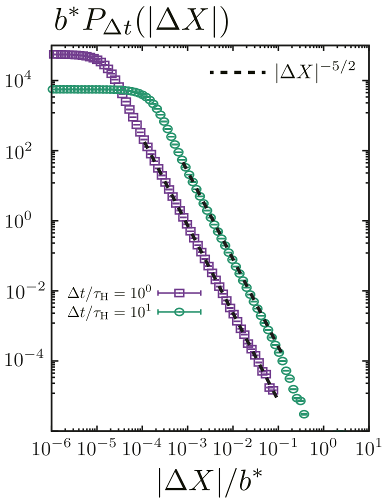
$$\Gamma\frac{d\boldsymbol{X}}{dt} = \boldsymbol{F}(t), \qquad \boldsymbol{F}(t) \equiv \sum_{i=1}^{N(t)}\boldsymbol{f_b}(t-\tau_i)$$

- Total Poisson intensity $\lambda \to \infty$ as $L \to \infty$
- Force shape function $\boldsymbol{f_b}(t)$: analytical form obtained up to 2nd order (non-closed loops)

Analytical approximation of force shape function



Tracer displacement distribution



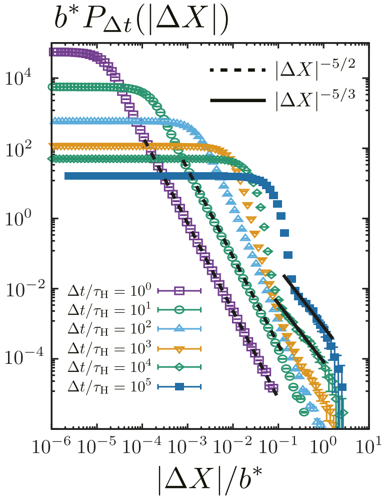
- Characteristic scales: $b^* \equiv \left(\frac{|p|}{\Gamma v_{\rm A}}\right)^{1/2}$

$$\tau_{\rm H} \equiv \frac{b^*}{v_{\rm A}}, \qquad \tau_{\rm C} \equiv \frac{1}{\rho v_{\rm A} \pi d^2}$$

- Scaling behaviour:

$$P_{\Delta t}(|\Delta X|) \propto \begin{cases} |\Delta X|^{-5/2} & \Delta t \ll \tau_{\rm H} \\ & \end{cases}$$

Tracer displacement distribution



- Characteristic scales: $b^* \equiv \left(\frac{|p|}{\Gamma v_A}\right)^{1/2}$

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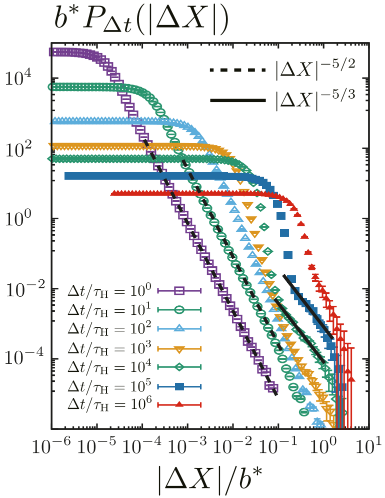
Tracer displacement distribution



- Characteristic scales: $$b^* \equiv \left(\frac{|p|}{\Gamma v_A}\right)^{1/2}$$

$$\tau_H \equiv \frac{b^*}{v_A}, \qquad \tau_C \equiv \frac{1}{\rho v_A \pi d^2}$$

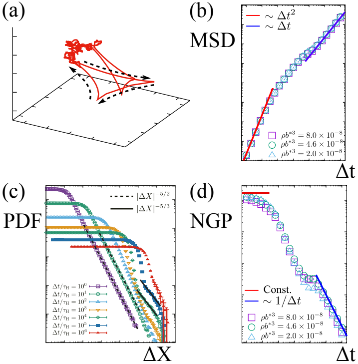
- Scaling behaviour:

$$P_{\Delta t}(|\Delta X|) \propto \begin{cases} |\Delta X|^{-5/2} & \Delta t \ll \tau_H \\ |\Delta X|^{-5/3} & \tau_H \ll \Delta t \ll \tau_C \\ e^{-\Delta X^2/2\sigma^2} & \Delta t \gg \tau_C \end{cases}$$

In 2D: $\alpha_H = 2, \quad \alpha_S = 4/3$

Tracer diffusion in active suspensions: Experimental results

Characteristic features		T1	T2	OT
Loopy trajectories	(a)	✓	✗	✓
Enhanced diffusion	(b)	✓	✗	✓
Non-Gaussian tails of the displacement statistics	(c)	✗	✓	✓
Reversion to Gaussian	(c)	✗	✓	✓
NGP $\simeq \Delta t^{-1}$	(d)	✗	✗	✓



Conclusions

- We derived a stochastic process that captures all empirical observations of the tracer dynamics
- First microscopic foundation of Lévy flight dynamics
- Process relates exponents of the power-law tails to the hydrodynamic interactions
  - Holtsmark regime: $\alpha_{\rm H} = 1 + D/\nu$, where ν is leading exponent in r
  - Scattering regime exponent $\alpha_{\rm S}$ depends on net displacement after one loop
- Crossover between Lévy flight and Gaussian regimes governed by time scale $\tau_{\rm C} \sim \frac{1}{\rho}$
  - Long-lived Lévy flight regime at low swimmer density
  - Localized diffusion at high swimmer density

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